Why is the Long-Run Tax on Capital Income Zero?  
Reinterpreting the Chamley-Judd Result*

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Abstract

Why is it optimal not to tax capital income in the long run in Chamley (1986) and Judd (1985)? This paper demonstrates that the answer follows from standard intuitions from the optimal commodity-tax literature. We show that the steady state assumption is critical for the Chamley-Judd result: in the steady-state, Engel curves for consumption become linear in labor earnings and consumption demands become equally complementary to leisure over time. From the optimal tax literature, we conclude that consumption should be taxed uniformly, which means that the optimal capital income tax is zero. We show that the intuition that capital income should not be taxed because the consumption distortions become infinite only applies when restrictions are imposed on the utility function. These restrictions ensure that consumption demands are equally complementary to leisure in the long run, thereby confirming standard optimal-tax intuitions. We also demonstrate that the optimal capital-income tax is zero irrespective of whether factor prices are determined in partial or general equilibrium. This result contradicts the intuition that optimal taxes on capital income are zero because the entire burden of capital income taxes is shifted to labor through general-equilibrium effects on factor prices.

JEL-code: H2
Key words: taxation of capital income, zero capital-income tax, Corlett-Hague motive, Chamley-Judd result

1 Introduction

Should capital income be taxed or not? This is one of the oldest and most important questions in public finance. However, the literature has not yet settled on a definite answer and the issue remains controversial from a policy perspective1. One of the main arguments against taxing capital income is provided by Chamley (1986) and Judd (1985), who developed neoclassical
growth models based on Ramsey (1928). These authors solved the Ramsey (1927) problem of deriving optimal proportional taxes on labor and capital income so as to generate a given amount of government revenue with the lowest possible distortions. They showed that in the long run, capital income should not be taxed and all revenue should be generated by taxing exclusively labor income. If capital income is to be taxed, it should be taxed in the short run only.

The work of Chamley (1986) and Judd (1985) has inspired a large literature which studied the conditions necessary for this puzzling result to hold. However, few have questioned the economic mechanisms that drive it or its economic intuition. We aim to clarify the intuition behind this result by employing standard explanations from the optimal commodity-tax literature. In doing so, we show how the previous literature has settled on somewhat misleading explanations, which we clarify.

Our argument is based on Corlett and Hague (1953), who demonstrate that commodity-tax differentiation is desirable when it alleviates the labor-supply distortions caused by the tax on labor income. Intuitively, when goods that are relatively more complementary to leisure are taxed at higher rates, individuals substitute away from leisure and work more. Hence, commodity-tax differentiation can alleviate distortions of the labor-income tax, but this comes at the expense of distorting commodity demands.\(^2\)

It has long been recognised that the dynamic optimal-tax problem is in fact a large Ramsey commodity-tax problem. Since a capital income tax is equivalent to a differentiated consumption tax where future consumption is taxed relatively more, we can readily apply the Corlett-Hague argument. Therefore, a capital income tax is only optimal when it alleviates the labor supply distortions caused by the labor income tax.

The main finding of this paper is that when the economy converges to a steady state, the Corlett-Hague taxation motive vanishes. In our model, the steady-state assumption forces all consumption Engel curves to become linear in labor earnings, irrespective of the utility function being used. This means that in the steady-state, consumption and leisure become equally complementary at all times. Since capital-income taxes generate distortions in saving and do not alleviate labor supply distortions, they should be set to zero in the long run.\(^3\)

We establish a close link between the zero-tax result in Chamley (1986) and Judd (1985) and the Atkinson and Stiglitz (1976) theorem for uniform commodity taxes under linear taxation. When consumption Engel curves are linear in labor earnings, proportional taxes on capital income impose the same labor-supply distortions as proportional taxes on labor income, but in addition also distort saving. Sandmo (1974) and Deaton (1979) derive that, in order to obtain linear Engel curves in consumption, the utility function must be 1) (weakly) separable between consumption and leisure and 2) (quasi-)homothetic in consumption. In the Chamley-Judd case,\(^2\)

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\(^2\)This is confirmed in Erosa and Gervais (2002) who use a OLG-version of the models in Chamley (1986) and Judd (1985). Jacobs and Broadway (2013) demonstrate that the Corlett-Hague intuition carries over to models with heterogeneous individuals, non-linear income taxation and linear commodity taxation.

\(^3\)Our paper is meant as a positive, methodological contribution aimed at clarifying the result of Chamley (1986) and Judd (1985) that capital income should not be taxed in the long run. It is not meant to serve as a normative policy prescription. Elsewhere, one of us has argued that capital income should be taxed at positive rates for various reasons that the framework of Chamley (1986) and Judd (1985) cannot address, see Jacobs (2013).
the steady-state assumption leads to linear Engel curves in consumption, regardless of the utility function. Thus, the standard requirements for no commodity-tax differentiation are met and taxes on capital income should optimally be zero.

By showing that standard optimal-tax principles underlie the zero tax on capital income, we reveal that standard intuitions for the zero long-run tax on capital income can be misleading. The first intuition in the literature is provided by Judd (1999) and subsequently used in Banks and Diamond (2010). These authors assert that the economy need not converge to a steady state for the optimal long-run tax on capital income to be zero. Since capital-income taxes impose an exponentially growing tax burden on consumption in the more distant future, it can never be optimal to set them to strictly positive rates in the long run. Such an explosive path of tax distortions in finite time is incompatible with standard Ramsey principles, which insist that tax distortions be smoothed out over time. Therefore, in order to rule out exponentially growing tax burdens, taxes on capital income should become zero in finite time.

We agree with Judd (1999) that the intuition for the zero capital-income tax result should be firmly rooted in optimal-tax principles. However, we also show that the simple Ramsey intuition alone is not sufficient to understand the mechanics of a capital income tax in the long run. The Ramsey tax-smoothing logic requires that consumption demands only depend on their own prices. Then, if price elasticities become constant, taxes on consumption should be perfectly smoothed out over time. We show that independent consumption demands are obtained only when strong restrictions are made on the utility function: separability over time and between consumption and leisure. This preference structure implies that in the long run, current and future consumption become equally complementary to leisure and the Corlett-Hague motive for taxes on capital income vanishes. As a result, there is no reason to distort intertemporal consumption choices to reduce distortions in labor supply and the optimal capital-income tax is zero in the long run. Hence, the Ramsey tax-smoothing intuition is a special case of the more general Corlett-Hague intuition that we pursue in this paper.

The second intuition is that in the steady-state all taxes on capital income will be shifted to labor due to general-equilibrium effects on factor prices (Auerbach and Kotlikoff, 1983; Correia, 1996; Mankiw, Weinzierl, and Yagan, 2009; Piketty and Zucman, 2013). In the steady-state the net return to capital is completely determined by exogenous factors such as the depreciation rate, the rate of time preference, the population growth rate and the rate of technological progress. Consequently, any tax on capital income results in a one-to-one increase of the gross return to capital so as to keep the net return constant. This requires a fall in the steady-state capital stock and leads to a corresponding drop in wages. As a result, the tax burden is completely shifted to labor. It is therefore concluded that it is better to tax labor income directly to avoid distortions in the capital market.

We show that this argument is misleading by analysing a partial-equilibrium version of the Ramsey model where we switch off any general-equilibrium effects on factor prices that could occur due to the taxation of capital income. We show that optimal-tax expressions in partial

\footnote{When compensated demands are independent, and only depend on their own price, then the compensated price elasticity of demand equals the the compensated cross elasticity of demand with respect to the wage rate. Hence, commodities that are more price elastic are more complementary to labor. See also Atkinson and Stiglitz (1980, Ch. 12).}
equilibrium are identical to those obtained in general equilibrium, thus confirming the results of Diamond and Mirrlees (1971) in a dynamic setting\(^5\). Therefore, the intuition why capital income should not be taxed in the long run cannot be that general-equilibrium effects in factor prices shift the entire tax burden towards labor. This contrasts heavily with the impressions that are given in Auerbach and Kotlikoff (1983), Correia (1996), Mankiw, Weinzierl, and Yagan (2009), and Piketty and Zucman (2013).

The current work aims to complement the meticulous analysis of \(\), who focused on the underlying assumptions of the Chamley-Judd result. These authors conclude that the original analysis of Chamley (1986) is only applicable when preferences are additively separable over time. In the other cases, the zero capital income tax is imposed on a zero tax base, or it coexists with a zero labor income tax, which is not the usual interpretation of the zero capital income tax. \(\) also criticize the results of Judd (1999), arguing that the assumptions made on the endogenous multipliers are analogous to assuming a zero capital income tax.

Our analysis focuses on what has become the canonical model for discussing the Chamley-Judd result, similar to \(\). This setting is close to Chamley (1986) and Judd (1999), but simplifies the analysis by assuming preferences that are additively separable through time. Thus, the first critique of \(\) does not apply. The second caveat is also not applicable, as our analysis of the work of Judd (1999) relates to the case when he does not make assumptions on the multiplier, but rather on the utility function.

The rest of the paper is structured as follows. In the next section, we introduce the general-equilibrium model and show that the consumption Engel curves are linear in income in the steady-state. In the third section, we show how our interpretation relates to the two intuitions in the literature. A final section concludes.

2 Long-run capital-income taxes in general equilibrium

2.1 Representative individual

This section starts with a general-equilibrium formulation of a closed economy as in Chamley (1986) and Judd (1985). There is an infinitely-lived representative individual who maximizes the discounted value of life-time utility:

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad u_{c_t}, u_{l_t} > 0, \quad u_{c_{c_t}}, u_{l_{l_t}} < 0.
\]  

The utility function \(u(c_t, l_t)\) in each period is increasing, strictly concave and twice differentiable in both consumption \(c_t\) and labor \(l_t\). The individual’s pure rate of time preference is captured by the discount factor \(\beta\).

The representative individual is endowed with an initial amount of assets \(a_0\) and one unit of time per period, which must be divided between work and leisure. Labor time in each period has to satisfy a time constraint: \(0 \leq l_t \leq 1\). The gross interest rate is \(r_t\) and the gross wage rate is \(w_t\). The government levies a proportional tax on capital income \(\tau^K_t\) and a proportional

\(^5\)This requires that the government is able to impose a 100% tax on pure profits if the production function does not exhibit constant returns to scale.
tax on labor income $\tau_L^t$ in every period. Consequently, the individual’s budget constraint is:

$$a_{t+1} = [1 + (1 - \tau_t^K) r_t] a_t + (1 - \tau_t^L) w_t l_t - c_t, \quad a_0 \text{ given},$$

(2)

$$\lim_{t \to \infty} \frac{a_{t+1}}{\prod_{s=0}^t [1 + (1 - \tau_s^K) r_s]} = 0.$$  (3)

Equation (3) says that the present discounted value of the individual’s terminal assets should be 0, thus ruling out explosive asset paths (a no Ponzi-scheme condition). By iterating the individual’s budget constraint forward and applying the transversality condition in equation (3), we obtain the individual’s life-time budget constraint:

$$\sum_{t=0}^\infty \frac{c_t}{\prod_{s=0}^t [1 + (1 - \tau_s^K) r_s]} = a_0 + \sum_{t=0}^\infty \frac{(1 - \tau_t^L) w_t l_t}{\prod_{s=0}^t [1 + (1 - \tau_s^K) r_s]}.$$  (4)

The net present value of life-time consumption should be equal to the sum of initial assets and the net present value of life-time labor earnings.

The representative individual’s problem consists of choosing her consumption $\{c_t\}_{t=0}^\infty$, labor supply $\{l_t\}_{t=0}^\infty$ and assets $\{a_{t+1}\}_{t=0}^\infty$ such that the life-time utility (1) is maximized subject to the budget constraint (4). Assuming an interior solution for $l_t$ and denoting the multiplier on period $t$’s budget constraint by $\beta \lambda_t$, we can obtain the first-order conditions that govern optimal labor supply and saving behavior:

$$u_{c_t} = \lambda_t$$

(5)

$$-\frac{u_{l_t}}{\lambda_t} = (1 - \tau_t^L) w_t,$$  

(6)

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = 1 + \frac{(1 - \tau_{t+1}^L) r_{t+1}}{\beta \lambda_{t+1}}.$$  (7)

Equation (6) shows that the individual should work until the marginal benefit of having one extra unit of leisure, $-u_{l_t}$, is equal to the loss in utility due to having less income, $u_{c_t} (1 - \tau_t^L) w_t$. The Euler equation (7) describes the optimal allocation of consumption across time: the individual should save until her increase in utility from consuming marginally more in the current period, $u_{c_t}$, is the same as her discounted increase in utility from investing that consumption increment at market prices and consuming it the next period, $\beta (1 + (1 - \tau_{t+1}^L) r_{t+1}) u_{c_{t+1}}$.

2.2 Government

The government’s objective is to maximize the representative individual’s utility, while satisfying an exogenous revenue requirement $g_t$ in every period. In what follows, we assume that the government can credibly commit to the policies it sets. We assume that the government can verify the aggregate capital and labor income, but has no access to lump-sum taxes. Thus, it can use two types of instruments to raise revenue: proportional taxes $\tau_t^L$ on labor income and $\tau_t^K$ on capital income, and issuance of debt $d_{t+1}$.

We assume that in period 0 the tax rate on capital income $\tau_0^K$ and the initial level of debt $d_0$ are given. Since we analyze a deterministic model without default, government bonds and

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6If the government were free to set the tax on period-0 capital, it would be taxing a completely inelastic good,
private assets are perfect substitutes. Perfect arbitrage thus ensures that the interest rate on government bonds equals the interest rate \( r_t \) on other assets. Hence, the period-by-period government budget constraint reads as:

\[
d_{t+1} = (1 + r_t)d_t + g_t - \tau_t^L w_t l_t - \tau_t^K r_t a_t,
\]

\[
\lim_{t \to \infty} \frac{d_{t+1}}{\prod_{s=0}^{t} (1 + r_s)} = 0.
\]

The government debt \( d_{t+1} \) also has to satisfy transversality condition (9) to rule out explosive paths for public debt.

### 2.3 Firms

There is one representative firm that uses capital \( k_t \) and labor \( l_t \) to produce output using a constant-returns-to-scale production technology \( f(k_t, l_t) \), which satisfies the Inada conditions and features positive and decreasing marginal returns in both capital and labor: \( f_t, f_k > 0, f_{tt}, f_{kk} < 0 \). Capital depreciates at rate \( \delta \). Profit maximization implies that marginal products equal marginal costs: \(^7\)

\[
f_k(k_t, l_t) = r_t + \delta,
\]

\[
f_l(k_t, l_t) = w_t.
\]

### 2.4 General equilibrium

Goods-market clearing requires that the total demand for goods – private consumption \( c_t \), public consumption \( g_t \), investment \( k_{t+1} - (1 - \delta)k_t \) – equal the supply of goods:

\[
c_t + g_t + k_{t+1} - (1 - \delta)k_t = f(k_t, l_t).
\]

Equilibrium in the capital market requires that the demand for capital by firms \( k_t \) and demand of government debt \( d_t \) equal the supply of assets by the representative individual \( a_t \):

\[
k_t + d_t = a_t.
\]

In order to ensure the existence of positive output at \( t = 0 \), we assume that the capital market is in equilibrium by definition in period 0: \( a_0 - d_0 = k_0 > 0 \). Moreover, the gross interest rate in period 0, \( r_0 \), is given and it is assumed to be consistent with capital-market clearing \(^8\).

### 2.5 Primal approach in general equilibrium

The government’s problem is to choose the sequence of taxes \( \{\tau_t^K, \tau_t^L\}_{t=0}^{\infty} \) that maximizes the representative individual’s life-time utility. In order to derive the optimal tax rules, we employ

\(^7\)There are no pure profits in equilibrium because returns to scale are constant.

\(^8\)Furthermore, the transversality condition for capital must hold: \( \lim_{t \to \infty} \frac{k_{t+1}}{\prod_{s=0}^{t} (1 + r_s)} = 0 \).
the primal approach to the optimal-tax problem. First, the government optimally derives the second-best allocation \(\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}\) subject to the resource and implementability constraints. Second, this allocation is decentralized using the tax instruments to obtain the same allocation as the outcome of a competitive equilibrium. An allocation is implementable when it satisfies Definition 1.

**Definition 1.** An allocation \(\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}\) is implementable with proportional taxes on capital and labor income if it satisfies the following conditions:

- There exists a sequence of taxes \(\{\tau^K_t, \tau^L_t\}\), input prices \(\{w_t, r_t\}_{t=0}^{\infty}\) and asset holdings \(\{a_{t+1}\}_{t=0}^{\infty}\) such that the allocation solves the individual’s problem, given the endowments and prices;
- There exist input prices \(\{w_t, r_t\}_{t=0}^{\infty}\), such that the firm maximizes its profits every period;
- The allocation satisfies the government budget constraint (8) every period;
- The allocation satisfies the aggregate resource constraint (12) every period;
- The allocation satisfies the domestic capital market equilibrium condition (13) every period.

The next step is to retrieve the implementability constraint, which is found as follows. First, use the individual’s first order conditions (6) and (7) to substitute out the net prices in the household budget constraint (2). Then, multiply the result by \(\beta^t u^c_t\), sum up over the individual’s entire lifetime and use the transversality condition for private assets (3):

\[
\sum_{t=0}^{\infty} \beta^t (c_t u^c_t + l_t u^l_t) = (1 + r^*_0) u^c_0 a_0 \equiv A(c_0, l_0, a_0, r^*_0). \tag{14}
\]

Lemma 1 shows that an allocation that satisfies the implementability (14) and aggregate resource constraints (12) is implementable with proportional taxes on capital and labor income. Therefore, instead of directly choosing the optimal taxes (the dual problem), we can solve the government’s problem by choosing the implementable allocation that maximizes the representative individual’s utility (the primal problem). We can then use the optimal allocation to retrieve the optimal tax rules.

**Lemma 1.** An allocation is implementable with proportional taxes if and only if it satisfies the implementability constraint (14) and the aggregate resource constraint (12).

**Proof.** See Appendix A.

\(\square\)

### 2.6 Optimal taxation

In order to simplify notation, we denote the multiplier on the implementability constraint by \(\theta\) and define pseudo utility function \(W(\cdot)\):

\[
W(c_t, l_t, \theta) \equiv u(c_t, l_t) + \theta (u^c_t c_t + u^l_t l_t). \tag{15}
\]
$W(c_t, l_t, \theta)$ can be interpreted as the net social value of private utility, while the multiplier $\theta$ is a measure of aggregate tax distortions.

Using the definition of $W(c_t, l_t, \theta)$ in equation (15), we can summarise the government’s problem as:

$$\max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \theta) - A_0,$$

subject to

$$k_0, r_0, \tau_k$$
given s.t. $a_0 - d_0 = k_0$, 

$$c_t + g_t + k_{t+1} - (1 - \delta)k_t = f(k_t, l_t),$$

$$\lim_{t \to \infty} \frac{k_{t+1}}{\prod_{s=0}^{t}(1 + r_s)} = 0.$$

We obtain the following first-order conditions for the government’s problem:

$$\frac{W_t}{W_c} = f_t = w_t,$$

(16)

$$\frac{W_c}{\beta W_{c_{t+1}}} = 1 + f_{k_{t+1}} - \delta = 1 + r_{t+1}.$$  (17)

Equation (17) is the counterpart of the individual’s first-order condition for labor supply (6). The government chooses the amount of labor in the economy until the social marginal utility cost of working $-W_l$ equals the social marginal benefit of working $w_t W_c$. Equation (16) is the government’s Euler equation for consumption, which is the counterpart of the individual’s Euler equation (7). The government chooses the consumption path such that the marginal decrease in social welfare incurred when saving in the current period $W_c$ is equal to the marginal increase in social welfare from consuming the proceeds of the savings in the next period $(1 + r_{t+1}) W_{c_{t+1}}$.

By taking derivatives of $W$ in (15), we can find expressions for $W_c$ and $W_l$:

$$W_c = u_{c_t}(1 + \theta + \theta \varepsilon^c_t),$$

(18)

$$W_l = u_{l_t}(1 + \theta + \theta \varepsilon^l_t).$$  (19)

Equations (19) and (18) defined the general-equilibrium elasticities $\varepsilon^c_t$ and $\varepsilon^l_t$:

$$\varepsilon^c_t \equiv \frac{u_{c_t} c_t}{u_{c_t}} + \frac{u_{c_t} l_t}{u_{l_t}} = \frac{\partial \ln u_{c_t}}{\partial \ln c_t} + \frac{\partial \ln u_{c_t}}{\partial \ln l_t},$$

(20)

$$\varepsilon^l_t \equiv \frac{u_{c_t} l_t}{u_{l_t}} + \frac{u_{c_t} c_t}{u_{c_t}} = \frac{\partial \ln u_{l_t}}{\partial \ln c_t} + \frac{\partial \ln u_{l_t}}{\partial \ln l_t}.$$  (21)

In what follows, we will focus on the general-equilibrium elasticity $\varepsilon^c_t$, which measures the distortions in consumption and labor supply with respect to consumption prices at time $t$. This elasticity in turn consists of two elasticities: the first, $\frac{\partial \ln u_{c_t}}{\partial \ln c_t}$, is the elasticity of the marginal utility of consumption with respect to $c_t$, which is a measure for the intertemporal elasticity of substitution in consumption. The marginal utility of consumption determines the saving behavior of the consumer, as seen in the Euler equation (7). The second term, $\frac{\partial \ln u_{c_t}}{\partial \ln l_t}$, is the elasticity of the marginal utility of consumption with respect to $l_t$. This elasticity captures the
complementarity of labor supply with respect to consumption decisions.

2.7 Zero long-run capital-income tax

In order to retrieve the optimal capital-income tax rate, we substitute the expression for $W_{ct}$ that we found into the government’s Euler equation (16):

$$\frac{u_{ct}}{\beta u_{ct+1}} \frac{(1 + \theta + \theta \varepsilon^c_t)}{(1 + \theta + \theta \varepsilon^c_{t+1})} = 1 + r_{t+1}. \tag{22}$$

Since every allocation optimally chosen by the government needs to also solve the individual’s problem, we can use the individual’s Euler equation (7) to find the optimal capital-income tax $r^K_{t+1}$:

$$\frac{r_{t+1} \tau^K_{t+1}}{1 + r_{t+1}} = \theta \frac{\varepsilon^c_t - \varepsilon^c_{t+1}}{1 + \theta + \theta \varepsilon^c_t}. \tag{23}$$

If the general-equilibrium elasticity of consumption today $\varepsilon^c_t$ is bigger (smaller) than the elasticity tomorrow $\varepsilon^c_{t+1}$, then it is optimal to tax (subsidize) capital income. This conforms to standard Ramsey principles; higher taxes should be levied at less elastic tax bases. Equation (23) provides the Chamley-Judd result of $\tau^K = 0$ in the steady state. In the steady-state, both consumption and leisure are constant, so the general-equilibrium elasticity $\varepsilon^c_t$ becomes constant over time. Since the numerator of the right-hand side becomes 0, the optimal tax on capital income is 0 in the long run.\(^9\)

2.8 Why is the long-run capital-income tax zero?

We argue that the same principles that underlie the desirability of uniform commodity taxation should also guide the desirability of not taxing capital income in the long run. In our model, a zero tax on capital income is equivalent to a uniform commodity tax on consumption at different dates. Corlett and Hague (1953) demonstrated that commodity-tax differentiation is generally desirable because introducing distortions in commodity demands helps alleviate distortions in labor supply. In particular, commodities that are more complementary to leisure should be taxed at relatively higher rates. In our setting, this implies that introducing saving distortions by taxing capital income is desirable when doing so reduces the distortions of labor taxation on labor supply.

The optimal-tax literature demonstrated that uniform commodity taxation is desirable only when commodity-tax differentiation does not help reduce labor market distortions. With linear tax instruments, this is the case only if the Engel curves of all commodities are linear in labor earnings, see also Sandmo (1974) and Deaton (1979). These authors show that linear Engel curves are obtained with utility functions that are weakly separable between commodities and labor and feature (quasi-)homotheticity of utility in consumption. In that case, commodity taxation is shown to be optimally uniform, since commodity taxes impose the same distortions.

\(^9\)Moreover, this result ensures that the transversality condition for government debt holds ex-post. Since $r = (1 - \tau^K) r$ when $\tau^K = 0$, the capital market equilibrium condition (13) holds and the transversality conditions for private assets and capital hold, the transversality condition for government debt will hold automatically.
on labor supply, but in addition also distort saving. The latter distortions can be avoided by only taxing labor income.

We obtain linear Engel curves of consumption demands in the steady-state of our model. In particular, suppose that the economy is in a steady-state from time $t = T$ onwards. Iterating the household budget constraint (2) forward from period $T$ onwards, and applying the transversality condition on private assets (3), we obtain the life-time budget constraint from time $T$ on:

$$\sum_{t=T}^{\infty} \frac{c_t}{\prod_{s=T}^{t} ((1 + (1 - \tau^K_s)r_s)} = a_T + \sum_{t=T}^{\infty} \frac{(1 - \tau^L_t)w_t l_t}{\prod_{s=T}^{t} ((1 + (1 - \tau^K_s)r_s)}.$$  \hspace{1cm} (24)

If the economy has settled down on a steady-state from period $T$ onwards, consumption $c_t$, labor $l_t$, assets $a_t$ and prices $(1 - \tau^K_t)r_t$ and $(1 - \tau^L_t)w_t$ are all constant for $t \geq T$. Moreover, in order to ensure the existence of a steady-state, the rate of time preference needs to be equal to the inverse of the discount rate, so $\beta = \frac{1}{1 + (1 - \tau^K)\tau}$.  

This means that the steady-state per-period household budget constraint (24) becomes:

$$c = (1 - \tau^K)r a + (1 - \tau^L)w l \iff c = a \beta + (1 - \tau^L)w l.$$  \hspace{1cm} (25)

Equation (25) shows that in the steady-state, the Engel curves for consumption become linear in labor earnings. Indeed, an increase in labor earnings $w l$ leads to a one-to-one increase in consumption $c$. Consequently, a capital income tax in the form of a linear tax on saving generates the same labor supply distortions as a revenue-equivalent linear tax on labor earnings. However, capital-income taxes also cause distortions in saving behaviour, which can be avoided by levying labor-income taxes only.

Therefore, the steady-state assumption forces all consumption Engel curves to become linear in labor earnings and consumption at all dates becomes equally complementary to labor supply. The zero capital-income tax is thus found for all utility functions, and not only weakly separable and (quasi-)homothetic ones.

### 3 Relation to the results in the literature

The optimal-tax literature discusses two main economic intuitions for the Chamley-Judd result that the tax on capital income should be zero in the long run. The first intuition is that a non-zero capital tax results in exploding tax distortions in finite time, which violates the Ramsey principle to smooth distortions over time, see also Judd (1999) and Banks and Diamond (2010).

The second intuition is that if the supply of capital is infinitely elastic in the long run, all taxes are borne by labor anyhow. Hence, it is better not to distort capital accumulation by setting a zero tax on capital income, see also Auerbach and Kotlikoff (1983), Correia (1996), Mankiw, Weinzierl, and Yagan (2009), and Piketty and Zucman (2013). This section argues that the first intuition can be interpreted as a special case of our generalized Corlett-Hague intuition and the second intuition is misleading.

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10 This result is obtained if one keeps $c_t$ constant in the Euler equation (7).
3.1 Exploding tax distortions in finite time?

Can the Chamley-Judd results be interpreted as a strict application of the Ramsey principle, as in Judd (1999) and Banks and Diamond (2010)? The answer is: only when restrictive assumptions are made on the utility function. As the expression for the optimal tax on capital income in equation (23) shows, the key element of the optimal capital-income tax is the general-equilibrium elasticity $\varepsilon^c_t$ of consumption and labor with respect to the consumption price. We can rewrite this elasticity as:

$$
\varepsilon^c_t = \frac{\partial \ln u_{c_t}}{\partial \ln c_t} + \frac{\partial \ln u_{c_t}}{\partial \ln \lambda_t} = \left( \frac{\partial \ln c_t}{\partial \ln \lambda_t} \right)^{-1} + \left( \frac{\partial \ln l_t}{\partial \ln \lambda_t} \right)^{-1},
$$

where $\lambda_t$ is the (shadow) price of consumption at date $t$.

Here, we see that the general-equilibrium elasticity of consumption consists of the (inverse) elasticity of consumption demand with respect to the price of consumption at time $t$, $\lambda_t \equiv u_{c_t}$, plus the (inverse) of the cross-elasticity of labor supply with respect to the price of consumption at time $t$. The elasticity $\varepsilon^c_t$ thus captures the distortion of changes in the consumption price on both consumption demand and labor supply at time $t$. The capital-income tax raises the price of consumption at date $t + 1$ relative to consumption at date $t$. Hence, it induces substitution away from future consumption and future leisure towards current consumption and leisure.

Taxes on capital income are desirable only when the aggregate elasticity today $\varepsilon^c_t$ is higher than the aggregate elasticity tomorrow $\varepsilon^c_{t+1}$. Equivalently, capital income should be taxed only when the combined distortions in consumption demand and labor supply tomorrow are lower than the combined distortions in both consumption and labor today. Naturally, when the utility function is separable, so that $\frac{\partial \ln l_t}{\partial \ln \lambda_t} = 0$, the Ramsey intuition is applicable. In this pure Ramsey case, capital income is taxed only when the elasticity of marginal utility of consumption $\sigma_t = \left( \frac{\partial \ln c_t}{\partial \ln \lambda_t} \right)^{-1}$ varies with time.

In the general case, the general-equilibrium elasticity $\varepsilon^c_t$ includes complementarities with labor, i.e. $\frac{\partial \ln l_t}{\partial \ln \lambda_t}$ that are not present in the own-price elasticities, i.e. $\frac{\partial \ln c_t}{\partial \ln \lambda_t}$. Therefore, the standard Ramsey intuition is not always applicable. By distorting the consumption prices, the capital-income tax not only distorts the intertemporal allocation of consumption, but also affects the intertemporal pattern of labor supply. Given that labor supply is distorted by the labor-income tax, a capital-income tax (or a subsidy) can be helpful to reduce labor-supply distortions. This depends on the specific pattern of $\frac{\partial \ln l_t}{\partial \ln \lambda_t}$ over time and no general conclusion can be drawn about this term without imposing further structure on the utility function.

The general-equilibrium elasticity $\varepsilon^c_t$ will be constant only in two cases: either when the economy is in a steady-state or when the utility function is strongly separable between consumption and leisure and homothetic in consumption. In the first case, the steady-state assumption forces consumption and leisure to be constant and the consumption Engel curves to be linear, as seen in Section 2.8 and in Chamley (1986) and Judd (1985). Therefore, in a steady-state, all the conditions for non-differentiated proportional consumption taxes are met regardless of preferences and taxes on capital income should then be zero.

Lemma 2 analyses the second case when capital income taxes are optimally zero. When the utility function is separable between consumption and labor (i.e. $u_{cl} = 0$) and the sub-utility
of consumption is homothetic, the elasticity $\sigma_t$ becomes constant and taxes on capital income are always zero. Consequently, the government cannot use capital-income taxes to reduce the distortions on labor supply.

**Lemma 2.** If the utility function is time separable and weakly separable between consumption and labor, with the consumption sub-utility being homothetic, then $\sigma_t$ is constant.

**Proof.** See ? and Appendix ??.

Judd (1999) argues that the optimal tax on capital income is zero in finite time even if the economy does not converge to a steady-state. He poses that the optimal tax on capital income would be driven down to zero as the deadweight loss of taxation would reach an upper bound in finite time. This finding seems to suggest that no restrictions on the utility function are needed to obtain a zero tax on capital income in finite time.

However, the analysis in Judd (1999) does not take into account the benefits of capital income taxation. While the wedge between the MRS between consumption at early periods and future consumption and the MRT can indeed grow at an exponential rate when capital income is taxed, so can the benefits derived from lowering the labor income distortions. Therefore, when the Corlett-Hague motive is present in finite time, the costs and the benefits of capital income taxation can grow at the same (exponential) rate. In such a case, a capital income tax is warranted on efficiency grounds, as long as the costs do not surpass the benefits. A similar argument is put forward in ?, who show that the ratio of the marginal costs and benefits of taxation remains constant.

Moreover, the analysis of Judd (1999) reveals that the deadweight loss of taxation becomes constant in finite time only if the general-equilibrium elasticity $\varepsilon^f_t$ converges to a constant in finite time, see his equation (28). Hence, his argument that distortions would reach an upper bound is equivalent to making the specific assumptions on the utility function that we identified to eliminate the Corlett-Hague motive for capital-income taxation.

Corollary (12) of Judd (1999) demonstrates that when these assumptions are violated, the optimal tax on capital income is not zero if the steady-state is not reached. In particular, assuming a Stone-Geary utility function that is separable between consumption and labor, Judd (1999) concludes that “the capital-income tax is never zero, but for reasons which are consistent with the inverse-elasticity rule”, i.e. the Ramsey rule. In this case, the first term (which we labeled $\sigma_t$) in equation (20) is never constant and the second term is 0. Thus, without invoking the steady-state assumption and without assuming (quasi-)homothetic preferences, the capital-income tax rate fluctuates according to the inverse of the elasticity of consumption. Note that the Ramsey intuition of taxing inelastic consumption demands at higher rates can be seen as a special case of the Corlett-Hague intuition. In particular, when consumption demands are independent, the goods that are less price elastic are also consumption goods that are relatively more complementary to leisure.

Therefore, the standard Ramsey intuition applied in Judd (1999) and Banks and Diamond (2010) need not always be applicable: this critically depends on whether the general-equilibrium elasticity $\varepsilon^f_t$ converges to a constant, which requires specific assumptions on the utility function or convergence to a steady-state.
3.2 Full tax shifting to labor?

Another common explanation for the zero optimal capital-income tax result can be found in the work of Auerbach and Kotlikoff (1983), Correia (1996), Mankiw, Weinzierl, and Yagan (2009) and Piketty and Zucman (2013). These authors assert that the supply of capital becomes infinitely elastic in the long run, so the entire burden of a capital-income tax is borne by labor. Since in the long run the net interest rate is fixed by exogenous factors such as the rate of time preference, population growth and depreciation, any decrease due to taxing capital income will be perfectly offset by a one-to-one increase in the gross interest rate.

Another common explanation for the zero optimal capital-income tax result can be found in the work of Auerbach and Kotlikoff (1983), Correia (1996), Mankiw, Weinzierl, and Yagan (2009) and Piketty and Zucman (2013). These authors assert that the supply of capital becomes infinitely elastic in the long run, so the entire burden of a capital-income tax is borne by labor. Since in the long run, the net interest rate is fixed by exogenous factors such as the rate of time preference, population growth and depreciation, any decrease due to taxing capital income will be perfectly offset by a one-to-one increase in the gross interest rate.

In this section, we show that this mechanism cannot be the driving force behind the Chamley-Judd result. We switch off the general-equilibrium effects by considering the case of an open economy. Since the gross interest rate is fixed in the world asset markets, the tax burden on capital cannot be shifted towards labor through general-equilibrium effects on factor prices.

To fix ideas, the representative individual and the government are allowed to access a perfectly competitive international capital market in which a foreign asset $x_t$ is traded. Foreign capital $x_t$ is supplied infinitely elastically and yields an exogenously given return $r_t$, which is the required return for private debt $a_t$ and government debt $d_t$. The optimization problems of the government and the representative individual are identical to closed-economy case, with the only difference that now both agents have access to the international capital market. Moreover, the implementability constraint remains identical to the one derived in (14).

The main difference with the general-equilibrium set-up stems from the firm’s behaviour. We now assume that the production technology $f(\cdot)$ does not employ domestic capital and only employs labor: $f(l_t) = A_t l_t$. In this way, we sever the link between wages and interest rates, while keeping everything else identical with the general-equilibrium setting. Profit maximization then implies that labor demand is perfectly elastic at the market wage: $A_t = w_t$.

In this open economy, total domestic production $w_t l_t$ need not equal domestic absorption $c_t + g_t$. Hence, the current account is determined by:

$$ c_t + g_t + x_{t+1} - (1 + r_t)x_t \leq w_t l_t, $$

$$ \lim_{t \to \infty} \frac{x_{t+1}}{\prod_{s=0}^{t}(1 + r_s)} = 0. $$

To prevent explosive paths of net foreign debt, we impose a no-Ponzi game condition: current account deficits are always repaid with later current account surpluses.

The capital-market equilibrium condition in this open-economy setting is very similar to the
general-equilibrium model in Section 2

\[ a_t - d_t = x_t. \]  

(29)

The left-hand side represents the demand for foreign capital: both the individual and the government demand assets in their intertemporal trades. The right-hand side of the equation represents the total supply of capital: foreign capital flows into the economy, which increases the intertemporal consumption possibilities compared to the case when the economy is closed.

Since the domestic firm does not employ capital in its production process, we need to modify the definition of an implementable allocation to include the flows of foreign investment \( x_t \) instead of \( k_t \), see Definition 2.

**Definition 2.** An allocation \( \{ c_t, g_t, x_{t+1} \}_{t=0}^{\infty} \) is implementable with proportional taxes on capital and labor income, given the input prices \( \{ w_t, r_t \}_{t=0}^{\infty} \), if it satisfies the following conditions:

- There exists the sequence of taxes \( \{ \tau^K_t, \tau^L_t \} \) and a sequence of asset holdings \( \{ a_{t+1} \}_{t=0}^{\infty} \) such that the allocation solves the individual’s problem, given the endowments and prices;
- The allocation satisfies the government budget constraint (8) every period;
- The allocation satisfies the aggregate resource constraint (27) every period;
- The allocation satisfies the domestic capital market equilibrium (29) every period.

Lemma 3 is the counterpart of Lemma 1 in partial equilibrium. It shows that, for an allocation to be implementable with proportional taxes in a partial-equilibrium setting, it need only be feasible (satisfy the aggregate resource constraint (27)) and satisfy the implementability constraint (14).

**Lemma 3.** An allocation \( \{ c_t, g_t, x_{t+1} \}_{t=0}^{\infty} \) is implementable with proportional taxes on capital and labor income, given the input prices \( \{ w_t, r_t \}_{t=0}^{\infty} \), if and only if it satisfies the implementability constraint (14) and the aggregate resource constraint (27).

**Proof.** See Appendix B.

Thus, the government’s optimal-tax problem becomes:

\[
\begin{align*}
\max_{\{c_t, g_t, x_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \theta) - A_0 \\
\text{subject to } & w_t, r_t \text{ given, } \forall t \\
& x_t, l_t \text{ given s.t. } a_0 - d_0 = x_0 \\
& c_t + g_t + x_{t+1} - (1 + r_t) x_t = w_t l_t, \forall t \\
& \lim_{t \to \infty} \frac{x_{t+1}}{\prod_{s=0}^{t}(1 + r_s)} = 0
\end{align*}
\]

(30)

From this formulation it is obvious that the two optimal-tax problems in partial and general equilibrium are mathematically identical. The only difference is cosmetic: the government
chooses the amount of private domestic capital $k_{t+1}$ in general equilibrium and the amount of foreign capital $x_{t+1}$ in partial equilibrium. We thus confirm Diamond and Mirrlees (1971): optimal-tax expressions are identical in partial and general equilibrium. As a result, the steady-state optimal capital-income tax expression will still lead to the Chamley (1986) and Judd (1985) result: $\tau^K = 0^{11}$.

We have demonstrated that the optimal long-run capital-income tax is zero both in general equilibrium and in partial equilibrium. This finding is not consistent with the notion that in the long run the capital-income tax is completely shifted to labor via general equilibrium effects on interest rates and wages, as the latter are absent – by definition – in partial equilibrium.

4 Conclusion

This paper tried to answer the question: why is the long-run capital-income tax zero in Chamley (1986) and Judd (1985)? We demonstrated that standard optimal commodity-tax principles underlie the zero capital-income tax result. In particular, the steady-state assumption forces Engel curves of consumption to become linear in labor earnings. This holds true even if preferences are non-separable between consumption and leisure and non-homothetic in consumption. Hence, consumption at different dates becomes equally complementary with leisure. Standard optimal-tax intuitions therefore underlie the zero optimal tax on capital income in the long run: capital-income taxes cannot be used to offset some of the distortions of labor-income taxes on labor supply. Moreover, the standard Ramsey intuition for zero capital-income taxes – exploding tax distortions in finite time – needs reconsideration as this intuition is applicable only when restrictions are made on the utility function that are identical to the restrictions required to have no commodity-tax differentiation. We also showed that general-equilibrium effects cannot be the main driver behind the long-run zero optimal capital-income tax, since we found that the optimal capital-income tax is zero also in partial-equilibrium settings, where there are no general-equilibrium effects on interest rates at all.

References


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11This result depends on the absence of pure profits (constant returns to scale in production) or the presence of a pure profit tax when returns to scale are not constant. See also Correia (1996).


A Proof of Lemma 1

Proof. We first prove that an implementable allocation satisfies the implementability constraint (14) and the aggregate resource constraint (12). Since an implementable allocation solves the household problem by definition, we can use the household’s optimality conditions, transversality condition for assets and the budget constraint to derive the implementability constraint. Furthermore, an implementable allocation satisfies the aggregate resource constraint by construction. Next, we prove that an allocation \( \{c_t, l_t, k_{t+1}\}_{t=0}^{\infty} \) that satisfies (14) and (12) is implementable. We can start by defining the input prices \( r_t \) and \( w_t \) such that the firm’s optimality conditions hold:

\[
\begin{align*}
  r_t &\equiv f_k(k_t, l_t) - \delta \\
  w_t &\equiv f_l(k_t, l_t)
\end{align*}
\]

Given the factor prices, we can use the household’s first-order conditions to define the proportional taxes \( \{\tau_L^t, \tau^K_{t+1}\}_{t=0}^{\infty} \) that implement the allocation \( \{c_t, l_t, k_{t+1}\}_{t=0}^{\infty} \):

\[
\begin{align*}
  \tau_L^t &\equiv 1 + \frac{u_l}{w_t u_c} \\
  \tau^K_{t+1} r_{t+1} &\equiv 1 + r_{t+1} - \frac{u_c}{\beta u_c+1}
\end{align*}
\]

Since we know the initial asset endowment \( a_0 \) and the paths of consumption and labor and the net prices of labor and future consumption, we can recursively define the private asset holdings \( \{a_{t+1}\}_{t=0}^{\infty} \) such that the household budget constraint (2) holds:

\[
\begin{align*}
  a_{t+1} &\equiv (1 + (1 - \tau^K_t) r_t + (1 - \tau^L_t) w_t l_t - c_t
\end{align*}
\]

Iterating the equation above forward and using the expressions for the net prices and the implementability constraint, we then obtain the transversality condition for assets (3). In order to prove that the allocation satisfies the government budget constraint, we subtract the household budget constraint (2) from the aggregate resource constraint (12) and use the linear homogeneity of the production function (constant returns to scale):

\[
\begin{align*}
  a_{t+1} - k_{t+1} &\equiv (1 + r_t)(a_t - k_t) + g_t - \tau^K_t r_t a_t - \tau^L_t w_t l_t
\end{align*}
\]

If we denote \( d_t \equiv a_t - k_t \), we obtain both the condition for capital-market clearing (13) and the government budget constraint (8), thus proving that the implication holds in both directions.

B Proof of Lemma 3

Proof. We first prove that an implementable allocation satisfies the implementability constraint (14) and the aggregate resource constraint (27). Since an implementable allocation solves the household problem by definition, we can use the household’s optimality conditions, transversality condition for assets and the budget constraint to derive the implementability constraint.
Furthermore, an implementable allocation satisfies the aggregate resource constraint by construction. Next, we prove that an allocation \( \{ c_t, g_t, x_{t+1} \}_{t=0}^\infty \) that satisfies (14) and (27) is implementable, given the input prices \( r \) and \( w \). The proof follows exactly the same steps as the one for the general equilibrium case in Appendix A: we use the household’s first order conditions to calculate the taxes \( \{ \tau^L_t, \tau^K_t \}_{t=0}^\infty \) that would implement the allocation. We then define the private asset path such that the household budget constraint (2) holds and use the implementability constraint (14) and the private optimality conditions (6) and (7) to prove that the transversality condition for private assets (3) holds. The last step of the proof involves subtracting the aggregate resource constraint (27) from the household budget constraint (2) and defining the government debt \( d_t = a_t - x_t \). This proves that both the condition for capital-market equilibrium (29) and the government budget constraint (8) hold.

C Proof Lemma 2

In this section, we assume that utility is time separable, strongly separable between consumption and leisure and homothetic in consumption:

\[
U = \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(l_t)),
\]

where \( u(c_t) \) is homothetic. By the separability of the utility function, we can rewrite the general equilibrium elasticity \( \varepsilon^c_t \) as:

\[
\varepsilon^c_t = \frac{u_{c_t} c_t}{u_{c_t}}
\]

It is easy to show that, since \( u \) is homothetic, \( \varepsilon^c_t \) is constant. By homotheticity of \( u \), we know that the following holds for any level of \( \alpha \):

\[
\frac{u_{c_t}(c_t)}{u_{c_{t+1}}(c_{t+1})} = \frac{u_{c_t}(\alpha c_t)}{u_{c_{t+1}}(\alpha c_{t+1})}
\]

Since the expression above can be treated as an identity, we can differentiate it w.r.t \( \alpha \) and set \( \alpha \) to 1:

\[
\frac{u_{c_t} c_t}{u_{c_t}} = \frac{u_{c_{t+1} c_{t+1}} c_{t+1}}{u_{c_{t+1}}}
\]

Since the expression above holds for any \( t \), we can conclude that \( \varepsilon^c_t \) is constant.